# Section 3: Signs of Operation and Comparison

### Operation signs:

| Braille | Print | Unicode | Unicode name, alternate names [description] |
| --- | --- | --- | --- |
| "6 | + | 002b | plus |
| "- | − | 2212 | minus [when distinguished from hyphen] |
| "8 | × | 00d7 | multiplication, times [cross] |
| "4 | ⋅ | 22c5 | dot operator, times [midline dot] |
| "4 | · | 00b7 | middle dot, times [midline dot] |
| "4 | ∙ | 2219 | bullet operator, times [midline dot]  |
| "/ | ÷ | 00f7 | division, divided by [horizontal line dotted above and below] |
| \_6 | ± | 00b1 | plus-minus, plus-or-minus [plus over minus] |
| \_- | ∓ | 2213 | minus-or-plus [minus over plus] |
| 3 | ∶ | 2236 | ratio, is to [colon] |
| "0 | ∘ | 2218 | ring operator, composite function, hollow dot [midline hollow dot] |
| "9 | ∗ | 2217 | asterisk operator, star [midline asterisk] |

### Comparison signs:

| Braille | Print | Unicode | Unicode name, alternate names [description] |
| --- | --- | --- | --- |
| "7 | = | 003d | equals |
| "7@: | ≠ | 2260 | not equal to [cancellation line through an equals] |
| @< | < | 003c | less-than |
| @> | > | 003e | greater-than |
| \_@< | ≤ | 2264 | less-than or equal to [bar under less-than] |
| \_@> | ≥ | 2265 | greater-than or equal to [bar under greater-than] |
| .@< | ≪ | 226a | much less-than [nest of two less-than signs] |
| .@> | ≫ | 226b | much greater-than [nest of two greater-than signs] |
| \_= | ≡ | 2261 | identical to, congruent to, equivalent to [three horizontal lines] |

| Braille | Print | Unicode | Unicode name, alternate names [description] |
| --- | --- | --- | --- |
| #\_l | ⦀ | 2980 | triple vertical bar delimiter, similar to [three vertical lines] |
| #\_l | ⫴ | 2af4 | triple vertical bar binary relation, similar to [three vertical lines] |
| #\_l | ⫼ | 2afc | large triple vertical bar operator [three vertical lines] |
| @9 | ∼ | 223c | tilde operator, similar to, varies with, difference between [midline tilde] |
| @9 | ~ | 007e | tilde [midline tilde] |
| \_9 | ≃ | 2243 | asymptotically equal to, approximately equal to [tilde over horizontal line] |
| ^9 | ≈ | 2248 | almost equal to, approximately equal to, asymptotic to [tilde over tilde] |
| "\_9 | ≅ | 2245 | approximately equal to, congruent to [tilde over equals] |
| ."7 | ≑ | 2251 | geometrically equal to, approximately equal to, equivalent to [equals dotted above and below] |
| ^"7 | ≏ | 224f | difference between, approximately equal to [equals with bump in top bar] |
| \_"7 | ∝ | 221d | proportional to, varies as [infinity sign open on the right] |
| 33 | ∷ | 2237 | proportion, as [two colons] |
| #l | ∥ | 2225 | parallel to [two vertical lines] |
| #- | ⟂ | 27c2 | perpendicular to, orthogonal to [vertical line meeting horizontal line] |

*Refer to:* Section 6, Fractions, for fraction lines; Section 10, Set Theory, Group Theory and Logic, for signs of operation and comparison used in those subjects; Section 13, Arrows, for arrows when used as signs of comparison; Section 17, Computer Notation for ASCII signs; and to *Rules of Unified English Braille*, Section 3.17, for use of signs of operation and comparison in literary material.

## 3.1 Spacing

*Note:* The presence or absence of spaces in braille is an important aid to parsing mathematical expressions and equations. Print spacing is often simply a matter of printing style.

3.1.1 In general, signs of operation are unspaced in braille and signs of comparison are spaced in braille.

*Refer to:* 3.1.2 through 3.1.6 for exceptions to this general guideline.

##### Examples:



"2y equals x plus 4"

#by "7 x"6#d



"5 minus 3 is not equal to 3 minus 5"

#e"-#c "7@: #c"-#e



"3 times 5 equals 5 times 3 equals 15"

#c"8#e "7 #e"8#c "7 #ae

200g×5 = 1kg

"200g times 5 equals 1kg"

#bjj;g"8#e "7 #akg



"Area equals bh equals 5 times [dot] 3 equals 15"

,>ea "7 bh "7 #e"4#c "7 #ae



"3 over 15 equals 3 divided by 15 equals 0.2"

#c/ae "7 #c"/#ae "7 #j4b



"15 plus or minus 0.5"

#ae\_6#j4e



"x squared minus y squared
equals (x plus or minus y)(x minus or plus y)"

x;9#b"-y9#b

 "7 "<x\_6y">"<x\_-y">



"the ratio 2 to 4 equals the ratio 6 to 12"

#b3#d "7 #f3#ab



"the ratio x to y"

x;3y or ;;x3y



"the ratio 1 is to 2 as [two colons] x is to 6"

#a3#b 33 x3#f



"3 times 1 is less than 3 plus 1"

#c"8#a @< #c"6#a



"4 squared is greater than 4 plus 4"

#d9#b @> #d"6#d



"Find theta if 0 is less than or equal to theta is less than or equal to pi"

,f9d .? if #j \_@< .? \_@< .p

If *a* ≤ *b*, then −*a* ≥ −*b*

"If a is less than or equal to b, then minus a is greater than or equal to minus b"

,if a \_@< ;b1 !n "-a \_@> "-b



"1 is much less than 1000000"

#a .@< #ajjjjjj



"1 is much greater than 0.0000001"

#a .@> #j4jjjjjja



"1 is congruent to [three horizontal lines] 5 modulo 4"

#a \_= #e"<mod #d">

ABCD ⦀ EFGH

"ABCD is similar to [three vertical lines] EFGH"

,,abcd #\_l ,,efgh



"x is equivalent to [tilde] y"

;x @9 ;y

△*ABC* ~ △*DEF*

"triangle ABC is similar to [tilde] triangle DEF"

;$#c:,,abc @9 ;$#c:,,def



"3.9 times 4.1 is approximately equal to [tilde over horizontal line] 16"

#c4i"8#d4a \_9 #af

15°C ≈ −9°F

"15 degrees C is approximately equal to [tilde over tilde] minus 9 degrees F"

#ae^j,c ^9 "-#i^j,f

*ABC* ≅ *XYZ*

"ABC is congruent to [tilde over equals sign] XYZ"

,,abc "\_9 ,,xyz

π ≑ 3.142

"pi is approximately equal to [equals sign dotted above and below] 3.142"

.p ."7 #c4adb

*x* ≑ *y* implies *y* ≑ *x*

"x is approximately equal to [equals sign dotted above and below] y implies y is approximately equal to [equals sign dotted above and below] x"

;x ."7 ;y implies ;y ."7 ;x

 (k, l) ≑ (m, n)

"(k, l) is equivalent to [equals sign dotted above and below] (m, n)"

"<;k1 ;l"> ."7 "<;m1 ;n">

or

;;;"<k1 l"> ."7 "<m1 n">;'

46×32 ≏ 50×30

"46 times 32 is approximately equal to [equals sign with bump in top bar] 50 times 30"

#df"8#cb ^"7 #ej"8#cj



"two-thirds is approximately equal to [equals sign with bump in top bar] 0.67"

#b/c ^"7 #j4fg



"v subscript 1 is approximately equal to [equals sign with bump in top bar] 0"

v;5#a ^"7 #j



"If y is proportional to x then y equals kx"

,if ;y \_"7 ;x !n ;y "7 kx



"PQ is parallel to KL and PQ is perpendicular to XY"

,,pq #l ,,kl & ,,pq #- ,,xy

3.1.2 Signs of operation may be spaced when they are first taught, before transitioning to normal spacing practice.

##### Examples:



"3 plus 5 equals 8"

#c "6 #e "7 #h



"8 minus 5 equals 3"

#h "- #e "7 #c

3.1.3 Signs of comparison are unspaced when they appear in an expression which is not on the base line.

##### Examples:



"the sum from n equals 1 to 10 of 3n"

,.s.5;<n"7#a>.9#aj#cn



"the integral from x equals 1 to x equals 5 of x squared dx"

;;!5<x"7#a>9<x"7#e>x9#b;dx

3.1.4 Signs of comparison may be unspaced in order to avoid dividing an expression between braille lines.

##### Example:



"(x plus 10)(x minus 4) is less than or equal to 7 x squared"

"<x"6#aj">"<x"-#d">\_@<#gx9#b

3.1.5 Signs of operation may be spaced to enhance the parsing of an expression which includes spaces between quantities and their units.

*Refer to:* 9.3 for spacing of signs of operation with function names and their arguments.

##### Examples:

2 ft + 4 ft = 6 ft

"2 ft plus 4 ft equals 6 ft"

#b ft "6 #d ft "7 #f ft

57.2 mm ÷ 10 = .572 cm

"57.2 mm divided by 10 equals .572 cm"

#eg4b mm "/ #aj "7 #4egb cm

3.1.6 Follow print spacing for signs of operation and comparison when the adjacent text is not a wholly mathematical expression.

##### Examples:

x axis from −4 to +5

"x axis from negative 4 to positive 5"

;x axis f "-#d to "6#e



"The answer is negative one half."

,! answ] is "-#a/b4

The area receives <2 inches of rain.

"The area receives less than 2 inches of rain."

,! >ea rcvs @<#b 9\*es ( ra94

a range of ≏4μg

"a range of approximately [equals sign with bump in top bar] 4 micrograms"

a range ( ^"7#d.mg

## 3.2 Minus "- -

*Note*: The print minus is very similar to the print hyphen but is normally shown as a slightly longer horizontal line, particularly in technical material.

3.2.1 It is permissible to use a braille hyphen for a minus which is indistinguishable from a hyphen in print.

##### Example:

current balance - initial balance = accumulated interest

"current balance minus [hyphen] initial balance equals accumulated interest"

curr5t bal.e - 9itial bal.e

 "7 a3umulat$ 9t]e/

## 3.3 Positive and negative numbers

3.3.1 Use a superscript indicator when a plus or minus, indicating a positive or negative number, is in the superscript position.

*Note:* It is permissible to describe the superscript position in a transcriber's note instead of using a superscript indicator.

##### Examples:



"Evaluate negative 2 plus negative 3 [with the minus signs in the superscript position in print]"

,evaluate ;9"-#b"69"-#c



"minus 2 degrees C or plus 2 degrees F [with the minus and plus signs in the superscript position in print]"

[open tn]Print uses a superscript
plus for a positive number and
a superscript minus for a
negative number. The
superscript position is not shown
in braille.[close tn]

@.<,pr9t uses a sup]script

 plus = a positive numb] &

 a sup]script m9us = a

 negative numb]4 ,!

 sup]script posi;n is n %[n

 9 brl4@.>

"-#b^j,c or "6#b^j,f

## 3.4 Hollow dot "0

3.4.1 Use the hollow dot to represent the mathematical sign of operation.

*Refer to: Rules of Unified English Braille*, section 3.5 for the bullet and section 3.11 for the degree sign.

##### Examples:



"(f plus g) hollow dot h equals f hollow dot h plus g hollow dot h"

"<f"6g">"0h "7 f"0h"6g"0h



"(f hollow dot g)(x) equals f(g(x))"

"<f"0g">"<x"> "7 f"<g"<x">">

## 3.5 Asterisk, star "9

3.5.1 Use the braille asterisk to represent the print midline asterisk used as a sign of operation in mathematics. It is generally brailled unspaced.

*Note:* Both the midline asterisk and the raised asterisk are represented by the same braille sign.

*Refer to:* Section 17, Computer Notation for the asterisk as an ASCII symbol; and *Rules of Unified English Braille*, section 3.3 for the asterisk used in literary material.

##### Examples:



"3 times [asterisk] 2 equals 2 times [asterisk] 3"

#c"9#b "7 #b"9#c

 is distributive over  if


"asterisk is distributive over hollow dot if
a asterisk (b hollow dot c)
equals (a asterisk b) hollow dot (a asterisk c)"

"9 is 4tributive ov] "0 if

 a"9"<b"0c">

 "7 "<a"9b">"0"<a"9c">