Section 3: Signs of Operation and Comparison

Operation signs:

Braille	Print	Unicode	Unicode name, alternate names [description]
⠐⠖	+	002b	plus
⠐⠤	−	2212	minus [when distinguished from hyphen]
⠐⠦	×	00d7	multiplication, times [cross]
⠐⠲	·	22c5	dot operator, times [midline dot]
⠐⠲	·	00b7	middle dot, times [midline dot]
⠐⠲	·	2219	bullet operator, times [midline dot]
⠐⠌	÷	00f7	division, divided by [horizontal line dotted above and below]
⠲⠖	±	00b1	plus-minus, plus-or-minus [plus over minus]
⠲⠤	∓	2213	minus-or-plus [minus over plus]
⠲	:	2236	ratio, is to [colon]
⠐⠖	∘	2218	ring operator, composite function, hollow dot [midline hollow dot]
⠐⠔	*	2217	asterisk operator, star [midline asterisk]

Comparison signs:

Braille	Print	Unicode	Unicode name, alternate names [description]
⠐⠶	=	003d	equals
⠐⠶⠈⠱	≠	2260	not equal to [cancellation line through an equals]
⠐⠣	<	003c	less-than
⠐⠜	>	003e	greater-than
⠸⠐⠣	≤	2264	less-than or equal to [bar under less-than]
⠸⠐⠜	≥	2265	greater-than or equal to [bar under greater-than]
⠸⠐⠣⠣	≪	226a	much less-than [nest of two less-than signs]
⠐⠜⠜	≫	226b	much greater-than [nest of two greater-than signs]
⠶⠿	≡	2261	identical to, congruent to, equivalent to [three horizontal lines]

Braille	Print	Unicode	Unicode name, alternate names [description]
⠸⠳⠸⠳⠸⠳	⦀	2980	triple vertical bar delimiter, similar to [three vertical lines]
⠨⠳⠸⠳⠸⠳	⫴	2af4	triple vertical bar binary relation, similar to [three vertical lines]
⠨⠳⠨⠳⠸⠳	⫼	2afc	large triple vertical bar operator [three vertical lines]
⠪⠒⠕	~	223c	tilde operator, similar to, varies with, difference between [midline tilde]
⠈⠔⠪	~	007e	tilde [midline tilde]
⠈⠔⠪	≃	2243	asymptotically equal to, approximately equal to [tilde over horizontal line]
⠠⠔⠪	≈	2248	almost equal to, approximately equal to, asymptotic to [tilde over tilde]
⠈⠔⠒⠪	≅	2245	approximately equal to, congruent to [tilde over equals]
⠐⠪⠒⠦	≑	2251	geometrically equal to, approximately equal to, equivalent to [equals dotted above and below]
⠐⠪⠪⠦	≏	224f	difference between, approximately equal to [equals with bump in top bar]
⠐⠪⠪⠦	∝	221d	proportional to, varies as [infinity sign open on the right]
⠄⠂⠠⠆	∷	2237	proportion, as [two colons]
⠨⠳⠸⠳	∥	2225	parallel to [two vertical lines]
⠨⠳⠠⠆	⊥	27c2	perpendicular to, orthogonal to [vertical line meeting horizontal line]

Refer to: Section 6, Fractions, for fraction lines; Section 10, Set Theory, Group Theory and Logic, for signs of operation and comparison used in those subjects; Section 13, Arrows, for arrows when used as signs of comparison; Section 17, Computer Notation for ASCII signs; and to *Rules of Unified English Braille*, Section 3.17, for use of signs of operation and comparison in literary material.

3.1　Spacing

Note: The presence or absence of spaces in braille is an important aid to parsing mathematical expressions and equations. Print spacing is often simply a matter of printing style.

3.1.1　In general, signs of operation are unspaced in braille and signs of comparison are spaced in braille.

Refer to: 3.1.2 through 3.1.6 for exceptions to this general guideline.

Examples:

$2y = x + 4$

"2y equals x plus 4"

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$5 - 3 \neq 3 - 5$

"5 minus 3 is not equal to 3 minus 5"

⠼⠑⠀⠐⠤⠀⠼⠉⠀⠈⠶⠈⠅⠀⠼⠉⠀⠐⠤⠀⠼⠑

$3 \times 5 = 5 \times 3 = 15$

"3 times 5 equals 5 times 3 equals 15"

⠼⠉⠀⠐⠦⠀⠼⠑⠀⠨⠅⠀⠼⠑⠀⠐⠦⠀⠼⠉⠀⠨⠅⠀⠼⠁⠑

200g×5 = 1kg

"200g times 5 equals 1kg"

⠼⠃⠚⠚⠛⠐⠦⠼⠑⠀⠨⠅⠀⠼⠁⠅⠛

Area = $bh = 5 \cdot 3 = 15$

"Area equals bh equals 5 times [dot] 3 equals 15"

⠠⠁⠗⠑⠁⠀⠨⠅⠀��⠀⠨⠅⠀⠼⠑⠀⠐⠲⠼⠉⠀⠨⠅⠀⠼⠁⠑

$$\frac{3}{15} = 3 \div 15 = 0.2$$

"3 over 15 equals 3 divided by 15 equals 0.2"

⠼⠉⠌⠁⠑⠀⠨⠅⠀⠼⠉⠀⠐⠌⠀⠼⠁⠑⠀⠨⠅⠀⠼⠚⠲⠃

15 ± 0.5

"15 plus or minus 0.5"

⠼⠁⠑⠀⠐⠖⠤⠀⠼⠚⠲⠑

$$x^2 - y^2 = (x \pm y)(x \mp y)$$

"x squared minus y squared
    equals (x plus or minus y)(x minus or plus y)"

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$$2:4 = 6:12$$

"the ratio 2 to 4 equals the ratio 6 to 12"

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$$x:y$$

"the ratio x to y"

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

$$1:2::x:6$$

"the ratio 1 is to 2 as [two colons] x is to 6"

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

$$3 \times 1 < 3 + 1$$

"3 times 1 is less than 3 plus 1"

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

$$4^2 > 4 + 4$$

"4 squared is greater than 4 plus 4"

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

Find θ if $0 \leq \theta \leq \pi$

"Find theta if 0 is less than or equal to theta is less than or equal to
    pi"

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

If $a \leq b$, then $-a \geq -b$

"If a is less than or equal to b, then minus a is greater than or equal to minus b"

⠐⠔ ⠁ ⠸⠪ ⠃⠂ ⠮⠝ ⠤⠁ ⠸⠕ ⠤⠃

1 ≪ 1000000

"1 is much less than 1000000"

⠼⠁ ⠸⠪⠪ ⠼⠁⠚⠚⠚⠚⠚⠚

1 ≫ 0.0000001

"1 is much greater than 0.0000001"

⠼⠁ ⠸⠕⠕ ⠼⠚⠲⠚⠚⠚⠚⠚⠚⠚⠁

$1 \equiv 5(\mathrm{mod}\ 4)$

"1 is congruent to [three horizontal lines] 5 modulo 4"

⠼⠁ ⠸⠿ ⠼⠑⠐⠣⠍⠕⠙⠀⠼⠙⠐⠜

ABCD ⦀ EFGH

"ABCD is similar to [three vertical lines] EFGH"

⠠⠁⠠⠃⠠⠉⠠⠙ ⠸⠈⠿ ⠠⠑⠠⠋⠠⠛⠠⠓

$x \sim y$

"x is equivalent to [tilde] y"

⠭ ⠈⠱ ⠽

$\triangle ABC \sim \triangle DEF$

"triangle ABC is similar to [tilde] triangle DEF"

⠫⠼⠇⠠⠁⠠⠃⠠⠉ ⠈⠱ ⠫⠼⠇⠠⠙⠠⠑⠠⠋

$3.9 \times 4.1 \simeq 16$

"3.9 times 4.1 is approximately equal to [tilde over horizontal line] 16"

⠼⠉⠲⠊⠐⠦⠼⠙⠲⠁ ⠈⠱⠿ ⠼⠁⠋

15°C ≈ −9°F

"15 degrees C is approximately equal to [tilde over tilde] minus 9
    degrees F"

⠼⠁⠑⠘⠨⠚⠠⠉⠀⠈⠔⠀⠐⠤⠼⠊⠘⠨⠚⠠⠋

ABC ≅ XYZ

"ABC is congruent to [tilde over equals sign] XYZ"

⠠⠁⠠⠃⠠⠉⠀⠘⠔⠐⠶⠀⠠⠭⠠⠽⠠⠵

π ≐ 3.142

"pi is approximately equal to [equals sign dotted above and below]
    3.142"

⠨⠏⠀⠐⠶⠻⠀⠼⠉⠲⠁⠙⠃

x ≐ y implies y ≐ x

"x is approximately equal to [equals sign dotted above and below] y
    implies y is approximately equal to [equals sign dotted above
    and below] x"

⠭⠀⠐⠶⠻⠀⠽⠀⠰⠔⠕⠍⠃⠇⠊⠑⠎⠀⠽⠀⠐⠶⠻⠀⠭

 (k, l) ≐ (m, n)

"(k, l) is equivalent to [equals sign dotted above and below] (m, n)"

⠐⠣⠅⠂⠀⠇⠐⠜⠀⠐⠶⠻⠀⠐⠣⠍⠂⠀⠝⠐⠜

or

⠰⠷⠅⠂⠀⠇⠰⠾⠀⠶⠐⠶⠻⠀⠰⠷⠍⠂⠀⠝⠰⠾

46×32 ≃ 50×30

"46 times 32 is approximately equal to [equals sign with bump in top
    bar] 50 times 30"

⠼⠙⠋⠈⠡⠼⠉⠃⠀⠐⠶⠈⠀⠼⠑⠚⠈⠡⠼⠉⠚

$$\frac{2}{3} \simeq 0.67$$

"two-thirds is approximately equal to [equals sign with bump in top bar] 0.67"

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$$v_1 \simeq 0$$

"v subscript 1 is approximately equal to [equals sign with bump in top bar] 0"

⠧⠰⠁⠀⠐⠶⠈⠲⠀⠀⠼⠚

If $y \propto x$ then $y = kx$

"If y is proportional to x then y equals kx"

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$PQ \parallel KL$ and $PQ \perp XY$

"PQ is parallel to KL and PQ is perpendicular to XY"

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3.1.2   Signs of operation may be spaced when they are first taught, before transitioning to normal spacing practice.

Examples:

$3 + 5 = 8$

"3 plus 5 equals 8"

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$8 - 5 = 3$

"8 minus 5 equals 3"

⠼⠓⠀⠤⠀⠼⠑⠀⠐⠶⠀⠼⠉

3.1.3   Signs of comparison are unspaced when they appear in an expression which is not on the base line.

Examples:

$$\sum_{n=1}^{10} 3n$$

"the sum from n equals 1 to 10 of 3n"

⠀⠐⠣⠩⠼⠁⠚⠩�⠼⠁⠐⠜⠀⠼⠉⠝

$$\int_{x=1}^{x=5} x^2 dx$$

"the integral from x equals 1 to x equals 5 of x squared dx"

⠮⠰⠦⠭⠀⠶⠼⠁⠀�⠭⠀⠶⠼⠑⠀⠭⠘⠆⠀⠙⠭

3.1.4   Signs of comparison may be unspaced in order to avoid dividing an expression between braille lines.

Example:

$$(x + 10)(x - 4) \le 7x^2$$

"(x plus 10)(x minus 4) is less than or equal to 7 x squared"

⠐⠣⠭�⠼⠁⠚⠐⠜⠐⠣⠭⠤⠼⠙⠐⠜⠀⠸⠣⠀⠼⠛⠭⠘⠆

3.1.5   Signs of operation may be spaced to enhance the parsing of an expression which includes spaces between quantities and their units.

Refer to: 9.3 for spacing of signs of operation with function names and their arguments.

Examples:

2 ft + 4 ft = 6 ft

"2 ft plus 4 ft equals 6 ft"

⠼⠃⠀⠋⠞⠀⠲⠀⠼⠙⠀⠋⠞⠀⠐⠶⠀⠼⠋⠀⠋⠞

57.2 mm ÷ 10 = .572 cm

"57.2 mm divided by 10 equals .572 cm"

⠼⠑⠛⠲⠃⠀⠍⠍⠀⠨⠒⠀⠼⠁⠚⠀⠐⠶⠀⠨⠢⠛⠃⠀⠉⠍

3.1.6   Follow print spacing for signs of operation and comparison when the adjacent text is not a wholly mathematical expression.

Examples:

x axis from −4 to +5

"x axis from negative 4 to positive 5"

⠀⠀⠀ (braille)

The answer is $-\frac{1}{2}$.

"The answer is negative one half."

⠀⠀⠀ (braille)

The area receives <2 inches of rain.

"The area receives less than 2 inches of rain."

⠀⠀⠀ (braille)

a range of ≏4μg

"a range of approximately [equals sign with bump in top bar] 4 microgram"

⠀⠀⠀ (braille)

3.2   Minus ⠀⠀

Note: The print minus is very similar to the print hyphen but is normally shown as a slightly longer horizontal line, particularly in technical material.

3.2.1   It is permissible to use a braille hyphen for a minus which is indistinguishable from a hyphen in print.

Example:

current balance - initial balance = accumulated interest

"current balance minus [hyphen] initial balance
        equals accumulated interest"

⠀⠀⠀ (braille)

3.3   **Positive and negative numbers**

3.3.1   Use a superscript indicator when a plus or minus, indicating a positive or negative number, is in the superscript position.

Note: It is permissible to describe the superscript position in a transcriber's note instead of using a superscript indicator.

Examples:

Evaluate $^-2 + {}^-3$

"Evaluate negative 2 plus negative 3 [with the minus signs in the superscript position in print]"

⠐⠣�A�A ⠃⠗⠁⠊⠇⠇⠑

$^-2°C$ or $^+2°F$

"minus 2 degrees C or plus 2 degrees F [with the minus and plus signs in the superscript position in print]"

[open tn]Print uses a superscript
    plus for a positive number and
    a superscript minus for a
    negative number. The
    superscript position is not shown
    in braille.[close tn]

3.4   **Hollow dot** ⠴⠲

3.4.1   Use the hollow dot to represent the mathematical sign of operation.

Refer to: Rules of Unified English Braille, section 3.5 for the bullet and section 3.11 for the degree sign.

Examples:

$$(f+g)\circ h = f\circ h + g\circ h$$

"(f plus g) hollow dot h equals f hollow dot h plus g hollow dot h"

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$$(f\circ g)(x) = f(g(x))$$

"(f hollow dot g)(x) equals f(g(x))"

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

3.5   Asterisk, star ⠔⠔

3.5.1   Use the braille asterisk to represent the print midline asterisk used as a sign of operation in mathematics. It is generally brailled unspaced.

Note: Both the midline asterisk and the raised asterisk are represented by the same braille sign.

Refer to: Section 17, Computer Notation for the asterisk as an ASCII symbol; and *Rules of Unified English Braille*, section 3.3 for the asterisk used in literary material.

Examples:

$$3*2 = 2*3$$

"3 times [asterisk] 2 equals 2 times [asterisk] 3"

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

* is distributive over ∘ if
$$a*(b\circ c) = (a*b)\circ(a*c)$$

"asterisk is distributive over hollow dot if
    a asterisk (b hollow dot c)
    equals (a asterisk b) hollow dot (a asterisk c)"

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

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